Technical Comments

Comments on "Recent Studies of the Laminar Base-Flow Region"

ARTHUR N. TIFFORD*

Nomenclature

C	=	const
h	=	static enthalpy
$H = h + u^2/2$	=	total enthalpy
k	=	dummy constant: 0 for rectilinear flows, 1 for axisymmetric flows
L	=	streamwise distance along body surface from forward stagnation point to base shoulder
r	=	radius arm of viscous layer (assumed independent of y)
u	=	streamwise velocity component
x	=	streamwise distance from body shoulder
y	=	transverse distance
μ	=	dynamic viscosity
ρ	=	density

Subscripts

c	=	recirculating core
e	=	outer edge of viscous layer

IT is generally recognized that Denison et al.[1, 2] have made some very interesting improvements on Chapman's separated flow analysis.[3–5] Their most recent publication on this subject,[6] however, is very misleading. For, whereas the results presented are purely qualitative, the profusive display of calculated data strongly suggests more inherent substance. Furthermore the qualitative conclusions that may be drawn with high confidence are directly obtainable from the governing basic differential equations and/or imposed boundary conditions without recourse to automatic computing machines. Those not so evident require a more refined flow model at the separation point and possibly in the inner wake core for a comparable confidence level to be achieved. The justification for the expenditure of time and money on the extensive computations described is therefore in doubt. The basis for these critical comments follows.

Accepting the limitations of the assumptions that $\rho\mu/\rho_e\mu_e = C$, the Lewis and Prandtl numbers are negligibly different from unity, all pressure gradients are negligible, etc., the authors found it convenient to transfer the shear-layer equations to the Crocco coordinate system. Thus,

Force Balance

$$u^*\partial F^*/\partial S^* = F^{*2}\partial^2 F^*/\partial u^{*2} \tag{1}$$

Energy Balance

$$u^*\partial H/\partial S^* = F^{*2}\partial^2 H/\partial u^{*2} \tag{2}$$

etc., where

$$u^* = u/u_e \tag{3}$$

$$S^* = S/S_W \tag{4}$$

$$S = \int_0^x C\rho_e\mu_e u_e r^{2k}dx \tag{5}$$

$$S_W = \int_{-L}^0 C\rho_e\mu_e u_e r^{2k}dx \tag{6}$$

$$F^* = FS_W^{1/2} \tag{7}$$

$$F = \partial u^*/\partial Y \tag{8}$$

$$Y = u_e r^k \int_0^y \rho dy \tag{9}$$

with the applicable boundary conditions

$$F^*(S^*, 0) = 0 \tag{10}$$

$$H(S^*, 0) = H_c \tag{11}$$

$$F^*(S^*, 1) = 0 \tag{12}$$

$$H(S^*, 1) = H_e \tag{13}$$

Two additional boundary conditions used, which are pertinent to the present discussion are 1) *assumed* momentum and thermal boundary-layer profiles at the body shoulder immediately after the freestream expansion and 2) the Chapman recompression hypothesis defining the stagnating base-flow streamline and thereby the end of the base-flow region.

The computational convenience offered by Eqs. (1) and (2) stems from the fact that they are in the form of the classical parabolic second-order differential equation representing time-variable heat conduction in a bar. From this point of view the S^* coordinate represents time, the u^* coordinate distance along the fictitious bar, and the quantity $(F^*)^2/u^*$ the effective local diffusivity. In fact, the whole boundary-layer development from the forward stagnation point onward may be analyzed, with suitable adjustments for the shoulder expansion at the rear, as the "associated heat-conduction" problem of a surface sheet of F^* shear sources and H energy sinks terminating abruptly at the location of the body shoulder. In any case, as the bar of interest travels over successively larger values of S^*, i.e., as time passes, the heat-conduction analogy directly provides the qualitative conclusions that are inherently valid. For example, since the expansion at the body shoulder increases the F^* levels there, the associated diffusivity is also increased, and a much more energetic dividing streamline is assured. A stronger trailing shock is therefore tolerable. It follows from Chapman's recompression hypothesis that the wake angle is enlarged, i.e., the base-flow region is shortened. On the other hand, mass transfer at the forward surface of the body reduces the F^* levels present at the start of the base-flow region and thereby also reduces the energization of the dividing streamline. It follows that the base-flow region is lengthened and cooled.

Wake cooling by mass transfer through the rear surface of the body also enlarges the flow region. From over-all physical and geometric considerations, it might be expected that the cooling effectiveness achieved is greater than when injecting a comparable mass of coolant through forward portions of the body surface. A quantitative comparison, however, requires a more precise description of the effect of the shoulder expansion upon the initial wake flow (and probably a better central-core representation) than that provided in Ref. 6.

Rather than agreeing with the authors that "the Chapman model has been carried about as far as possible. The worth of the theory in its present form will have to be decided when definitive base-flow data become available."[6] It seems to this writer that the theoretical (quantitative) potential of the improved Chapman model has not been tapped as yet.

Received October 15, 1964.

* Consultant. Associate Fellow Member AIAA.

References

[1] Denison, M. R. and Baum, E., "Compressible free shear layer with finite initial thickness," AIAA J. 1, 342–349 (1963).

[2] Baum, E., "Initial development of the laminar separated shear layer," AIAA J. 2, 128–131 (1964).

[3] Chapman, D. R., "Laminar mixing of a compressible fluid," NACA TN 1800 (1950).

[4] Chapman, D. R., "An analysis of base pressure at supersonic velocities and comparison with experiment," NACA TN 2137 (1951).

[5] Chapman, D. R., "A theoretical analysis of heat transfer in regions of separated flows," NACA TN 3792 (1956).

[6] Baum, E., King, H. H., and Denison, M. R., "Recent studies of the laminar base-flow region," AIAA J. 2, 1527–1534 (1964).

Reply by Authors to A. N. Tifford

ERIC BAUM,* HARTLEY H. KING,† AND M. RICHARD DENISON‡

Electro-Optical Systems, Inc., Pasadena, Calif.

IT is often true that qualitative conclusions may be obtained through judicious examination rather than detailed solutions of a mathematical problem. Where a problem involves many interdependent factors and is also nonlinear in character, some caution must be exercised in drawing sweeping conclusions. That such pitfalls are open even to those who are well versed in this art, and who have in addition the benefit of the hindsight offered by existing calculations, is evidenced by the examples treated in the preceding comment.

For instance, the preceding comment states that "since the expansion at the body shoulder increases the F^* levels there, the associated diffusivity is also increased, and a much more energetic dividing streamline is assured," a conclusion drawn from what must be considered a careful appraisal of the mathematical problem. However, the solution of the problem[1] indicates otherwise. The total enthalpy build-up along the dividing streamline is virtually unaffected by profile distortion caused by expansion. On the other hand, the dividing streamline velocity is sensitive to the expansion. It is clear that the increased diffusivity alone is not an adequate basis for prediction, since both the momentum and energy equations have identical diffusivities.

There is another, and perhaps overriding reason, why numerical calculations rather than order-of-magnitude estimates are appropriate for the prediction of observables from hypersonic wakes. In contrast to ordinary applications of heat transfer and fluid mechanics, observables are extremely sensitive to the detailed distribution of fluid properties. A 15% difference in neck enthalpy can produce an order-of-magnitude change in electron density.[2]

Some more or less obvious reasons for carrying out calculations can be added. One is data correlation. For this purpose it is desirable to have quantitative predictions based upon the best theoretical model available. Another use of the numerical results is for purposes of comparison with more approximate methods, such as moment methods. Finally, the experience gained with the numerical techniques has been helpful in further improving the flow model to include such features as the recirculating flow and viscous recompression, which have resisted analysis for many years.

In conclusion, we agree that extensive numerical calculations should be avoided whenever possible. More strongly, however, we feel that technical problems must be treated in a competent manner by whatever means are appropriate. In the present case the nature of the problem and the desired objectives make numerical analysis essential.

References

[1] Baum, E., King, H. H., and Denison, M. R., "Recent studies of the laminar base-flow region," AIAA J. 2, 1527–1534 (1964).

[2] Fernandez, F. L., Carson, J. L., and Anderson, D. A., "Wake radar cross section of slender re-entry vehicles," Aerospace Corp., Rept. TDR-469 (S5855-30)-1 (October 1964).

Received March 20, 1964; revision received January 18, 1965.

* Senior Scientist, Fluid Dynamics Department. Member AIAA.

† Senior Engineer. Member AIAA.

‡ Manager, Gas Dynamics Department. Member AIAA.

Comment on "A Transient Solution of the Fokker-Planck Equation"

YUN-YUAN SHI*

Carnegie Institute of Technology, Pittsburgh, Pa.

IN a recent paper, Eaton[1] claimed that he obtained a complete transient solution of the distribution function for the Fokker-Planck equation when the collision frequency is velocity-independent, corresponding to the Maxwell law of interaction. He also mentioned an earlier paper by Osipov.[2] The purpose of this note is to point out the fact that Eaton failed to see that his complete solution can easily be obtained from a corresponding linear equation obtained by Osipov by a single step of integration, and all these complications caused by Eaton were unnecessary. The author also wants to point out that the nonlinear differential equation obtained by Eaton is actually due to his improper choice of variable.

Following Eaton's notation, the corresponding linear differential equation obtained by Osipov is

$$dT_e/dt = -2\nu m/M(T_e - T) \tag{1}$$

where ν is collision frequency, m is electron mass, M is molecular mass, T_e is electron temperature, and T is gas temperature. This equation is valid as long as T and ν are velocity-independent. Since this is a first-order linear equation, the solution can easily be obtained by direct integration by use of an integrating factor.[3] The result is

$$T_e = \exp(-\beta\textstyle\int\nu dt)\{\int\beta T\nu \times [\exp(\beta\int\nu dt')]dt + K\} \tag{2}$$

where K is an integration constant and $\beta = 2m/M$. This is essentially the complete solution obtained by Eaton. Eaton failed to understand this simple extension. In addition, by choosing an improper variable $A = m/2kT_e$, he obtained the following new nonlinear differential equation:

$$(M/m\nu)\ (dA/dt) + 4kT/mA^2 - 2A = 0 \tag{3}$$

He then solved the foregoing equation and claimed that he obtained a solution for the problem. Actually all these complications are not necessary. If one substitutes $A = m/2kT_e$ into Eq. (3), one can see that Eq. (1) obtained by Osipov is easily reproduced.

References

[1] Eaton, C. F., "A transient solution of the Fokker-Planck equation," AIAA J. 2, 2033–2034 (1964).

[2] Osipov, D. I., "Conservation of the form of the Maxwellian distribution in a relaxing gas," AIAA, J. 1, 261–262 (1963).

[3] Martin, W. T. and Reissner, E., *Elementary Differential Equations* (Addison-Wesley, Inc., 1961) 2nd ed., Chap. 2, p. 60.

Received December 14, 1964. This study is performed under the National Science Foundation Research Grant No. GP-3040.

* Assistant Professor, College of Engineering and Science. Member AIAA.